

## Design of bridge vibration measurement system with MPU6050 and estimation of logarithmic decrement of bridge vibration in the presence of amplitude beat

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Published in  
VOI- 1 Issue: 4

DOI:10.5281/zenodo.17897006

PP: 13-25

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### Abstract

The vibration characteristics of bridge structures can be used as evaluation indicators for accurately diagnosing their deterioration. Damage detection methods of existing bridge structures also require effective management methods to reduce labor and their maintenance costs. In this paper, a digital three-axis acceleration sensor MPU6050 capable of measuring static (gravity) and dynamic (vibration) accelerations based on MEMS devices can be operated in a simple sensor network, a low cost, low weight and small size measurement system is designed and evaluated for the vibration characteristics of the bridge. We also consider the logarithmic damping characteristic for vibrations, one of the important dynamic parameters characterizing the bridge behavior. The logarithmic decrement for vibrations is usually evaluated by the vibration recording by digital signal processing. Among the processing procedures band-pass filtering is commonly used to extract spectral components of interest. We analyze the case when two very close spectral components are present in the acceleration records. We show that construction of the filter is non-trivial in such a situation and failing to choose optimal filter's parameters leads to significant logarithmic decrement estimation errors. Therefore we propose a method that does not require band-pass filtering but utilizes knowledge of amplitude beat presence. Simulation of the method indicated errors by an order less than using filtering.

**Keywords:** Logarithmic decrement; Amplitude beat; MPU6050

## Introduction

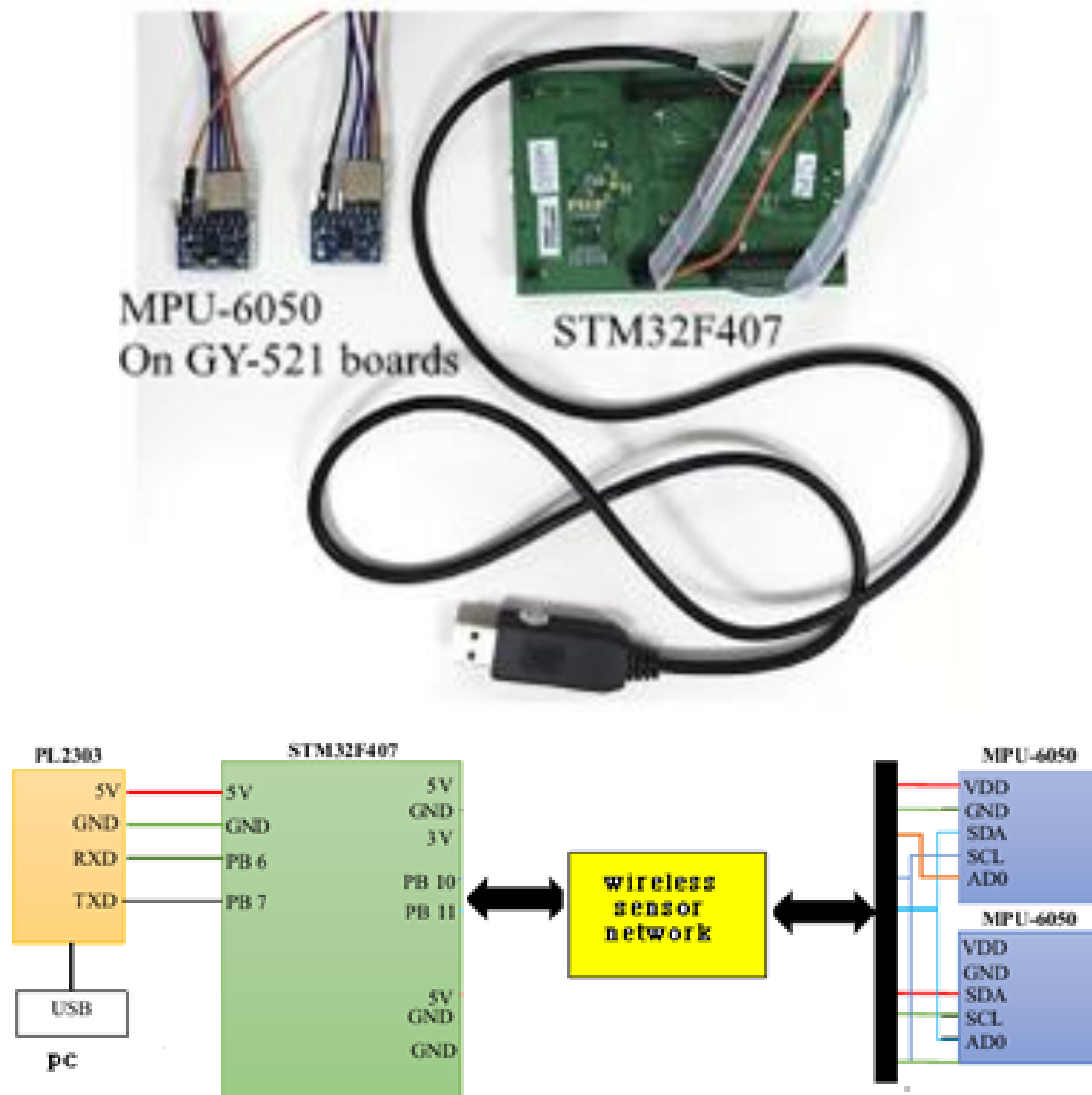
Because of MEMS technology advantages, it has today many industrial applications. This technology is employed in manufacturing of different sensors, like accelerometers, magnetometers, gyroscopes, flow and pressure sensors. The accelerometers manufactured in MEMS technology can measure both static acceleration (gravity), as well as dynamic vibrations.

Most of the parameters of this sensor can be configured in the software level, by writing to control registers via SPI or I2C interface.

The MEMS device is controlled by microcontroller.

The blocks of samples of measured acceleration in X, Y, Z axes are stored in RAM buffer. There is also an option of nonvolatile storage in FLASH memory.

Up to eight sensors are connected to a simple network.



**Figure 1. Proposed system diagram**

Dynamic behavior monitoring is vital for the structural integrity assessment and maintenance planning of bridges or other civil engineering structures. Dynamic characteristics like logarithmic decrement, natural frequencies, damping ratio, dynamic amplification factor are among most often mentioned through scientific studies and testing reports[1–5].

To give an example we have found out that logarithmic decrement

(LD) and natural frequencies are usually specified with the resolution of 0.01[3–5]. Avoidance of presenting estimation reliability could be related to the variety of processing steps that vibration records undergo before dynamic properties extraction.

The measurement uncertainty of LD estimation will also be influenced by amplitude and time measurement uncertainties despite the used processing method.

In this paper, we will focus only on the processing method specific errors.

Having in mind high precision and accuracy parameters of modern data acquisition systems we believe the influence of mentioned measurement uncertainties will be much less than the errors caused by the re-corded data processing method.

Sampled signatures of acceleration or displacement sensors are subject to baseline correction, integration, filtering, envelope or peak detection, ensemble averaging (in random decrement technique [6]), least squares fitting and other processing procedures that influence final estimated values of the dynamic properties. The influence of digital filter type (Butterworth, Chebyshev, Bessel) and their parameters (filter order, pass-band ripple, cutoff frequencies) upon strong-motion seismic parameters was examined and reported in [7]. It was found that some parameters interesting for engineering applications, e.g. peak ground acceleration, areas intensity, etc., are substantially affected by the filter applied.

Similarly in the tasks of bridge structural integrity assessment we may intuitively expect that the value of the measurement result is significantly dependant on the processing of raw data. Therefore, in this research we will investigate errors of logarithmic decrement estimation and how they are affected by the estimation procedures.

Blur is a general image degradation caused by low quality cameras or intentional photographing for highlighting moving or salient objects <sup>[1]</sup>.

The boundary blur information of ROI in an image can be used for image quality estimation, as well as, it has various information. For example, in medical image, the boundary blur information of ROI can reflect the stage of cancer or various diseases and can be applied to automatically focus on the field of vision distance measurement. Therefore, various methods for estimating the boundary blur degree in images have been developed worldwide and have been actively applied to image quality estimation and various applications. Y. Zou et al. proposed a robust image blur classifier to classify images into sharp, intentional blur, and unintentional blur. They used spatial pyramids to extract global features of the image, and then detected unintentional blur pixels by applying random forest <sup>[1]</sup>. In [2], the image-moment-based blur invariant features are calculated to eliminate the image motion blur and defocus blur that occur when there is a relative motion between the imaging camera and the detected object, and is used for accurate recognition of the object. In [3], a sharpness metric based on LBP and segmentation algorithm is proposed to identify the focused or defocused images, and used for image classification. In [4], the boundary from video surveillance was detected by improving the original Sobel operator certainly, and based on the blur degree of the recorded video, the image properties were obtained. In addition, several methods have been proposed by many researchers to estimate the boundary blur of an image, but in most cases, they have been used to remove the boundary blur information in an image. However, boundary blur can be used as a physical quantity that reflects a certain state of an object. For example, in the medical field, ROI blur in CT images may be used in the stage of a disease, and the boundary blur degree due to defocus in infrared image with optical systems can be effectively used for autofocusing and vision based distance measurement.

In this paper, two boundary detection algorithms were used to perform image segmentation to estimate the boundary blur degree of the ROI in the obtained image, and we estimated the blur degree of the ROI through comparison between the segmented results. For this, we first applied an image segmentation method using SLIC superpixels with relatively high accuracy of image segmentation, which uses segmenting and merging between superpixels to extract the boundary of the ROI and, based on it, apply a morphological algorithm to extract the final ROI. Then, a Level-Set algorithm with good global convergence

was applied to the original image to obtain the ROI, and estimate the boundary blur degree of the ROI using the difference of the two image segmentation results.

## 2. Problem description

The acceleration records were acquired on the investigated two-girder bridge having 18 m length. Sampling frequency of data acquisition system was 200 Hz. The acceleration waveform after band-pass filtering with Butterworth digital filter (pass-band frequencies from 3 Hz to 15 Hz, attenuation in stop-band is 20 dB) is shown in Fig. 2. Its power spectrum (Fig. 2) exhibits two close spectral components – 8.0 Hz and 8.9 Hz. We call F1 the main harmonic, F2 the adjacent harmonic and  $\Delta F = F2 - F1$  the frequency distance between harmonics.

The origin of the adjacent harmonic is not absolutely clear, but a possible explanation is that it may appear due to the vibration of the second girder (not the one used to attach the sensor). Another suggestion is that F2 harmonic is caused by the vibrations non-perpendicular to the main accelerometer axis. Presence of two very close spectral components results in amplitude beat in the time domain waveform.

The response of under damped Single Degree of Freedom (SDOF) system vibrations can be expressed according to [8]

$$x_1(t) = X_1 e^{-\xi_1 \omega_{n1} t} \cos(\omega_{d1} t + \theta_1) \quad (1)$$

Where  $X_1$  is the initial amplitude,  $\xi_1$  is the damping ratio,  $\omega_{n1} = 2\pi F1$  is the natural frequency,  $\theta_1$  is the damped frequency,  $\omega_{d1}$  is the oscillation phase.

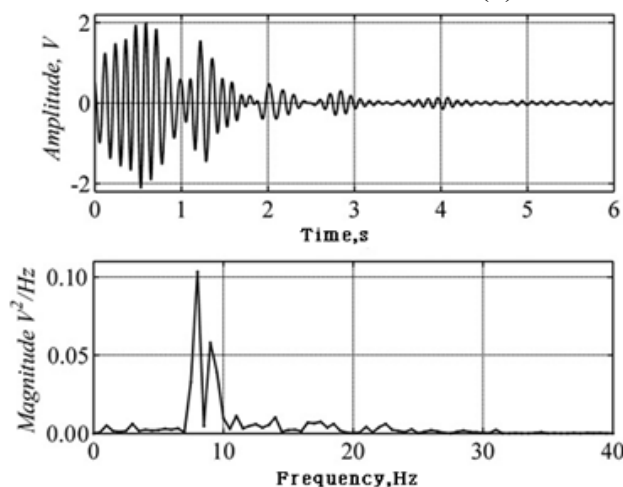
Since  $\omega_{d1} = \omega_{n1} \sqrt{1 - \xi_1^2} \approx \omega_{n1}$ , we later assume  $\omega_1 = \omega_{d1} = \omega_{n1}$ .

A similar equation can be written for the other frequency component (F2) response. Let us note it by  $x_2(t)$ . After minor arithmetical manipulations having in mind above mentioned assumption

$\omega_2 = \omega_{d2} = \omega_{n2}$ , the sum of  $x_1(t)$

and  $x_2(t)$  can be expressed as

$$\begin{aligned} x(t) &= x_1(t) + x_2(t) \\ &= X_1 e^{-\xi_1 \omega_1 t} [\cos(\omega_1 t + \theta_1)] + \\ &\quad (X_2 / X_1) e^{(\xi_1 \omega_1 - \xi_2 \omega_2) t} (\cos(\omega_2 t + \theta_2)) \end{aligned} \quad (2)$$



**Figure 2. Acceleration record in time domain and its power spectrum having two close frequency**

### components.

Let us now consider a boundary situation when

$$\xi_1 \omega_1 - \xi_2 \omega_2 \approx 0, \quad X_1 \approx X_2$$

Then Eq.(2) can be rewritten as

$$x(t) = 2X_1 e^{-\xi_1 \omega_1 t} \cos\left(\frac{\omega_2 - \omega_1}{2} t\right) \cos\left(\frac{\omega_2 + \omega_1}{2} t\right) \quad (3)$$

or rearranged to

$$x(t) = E(t) \cos(\omega_c t) \quad (4)$$

Where  $\omega_c = (\omega_2 + \omega_1) / 2 \approx \omega_1 = 2\pi F_c$  is the central frequency,  $E(t)$  is the slowly varying amplitude (envelope)

$$E(t) = 2X_1 e^{-\xi_1 \omega_c t} \cos(\omega_A t) \quad (5)$$

The amplitude beat frequency is noted by  $\omega_A = (\omega_2 - \omega_1) / 2 = 2\pi f_A$ .

The envelope can be calculated using either Hilbert transform or spline approximation.

The amplitude beat superimposed on the exponentially decaying response introduces certain difficulties for LD estimation from the time domain waveform. Indeed, decrement estimation procedure usually is carried out in three steps:

Band-pass filtering to damp all frequencies except spectral component of the interest.

Detection of oscillation peaks magnitudes  $x_p$  (see Fig. 3).

Calculation of the LD according to the expression

$$\delta_m = \frac{1}{k} \ln \frac{x_{p(n+k)}}{x_{p(n)}} \quad (6)$$

Where  $x_{p(n)}$  and  $x_{p(n+k)}$  are amplitudes of the  $n$ -th and the  $(n+k)$  -th peak correspondingly. Another widely used LD evaluation procedure is based on the least squares fit of the response function

$$y(t) = b_0 e^{-b_1 t} \quad (7)$$

using reference points  $(t_{D(i)}, x_{D(i)}, i = \overline{1, N})$ , where  $tp(i)$  is the time moment of the  $i$ -th peak  $x_{p(i)}$  (see Fig. 3),  $N$  is the number of peaks used.

According to Eq.(1), the exponent coefficient  $b_1$  is expressed  $b_1 = \xi_1 \omega_1 = (\delta_m / 2\pi) \cdot (2\pi F_1)$ , and therefore LD can be calculated as  $\delta_m = b_1 / F_1$ .

Plotting filtered acceleration records in time domain reveals that the errors of LD measurement will be significant due to the influence of the spectral component  $F_2$  which cannot be damped sufficiently because of its vicinity to the main frequency  $F_1$ .



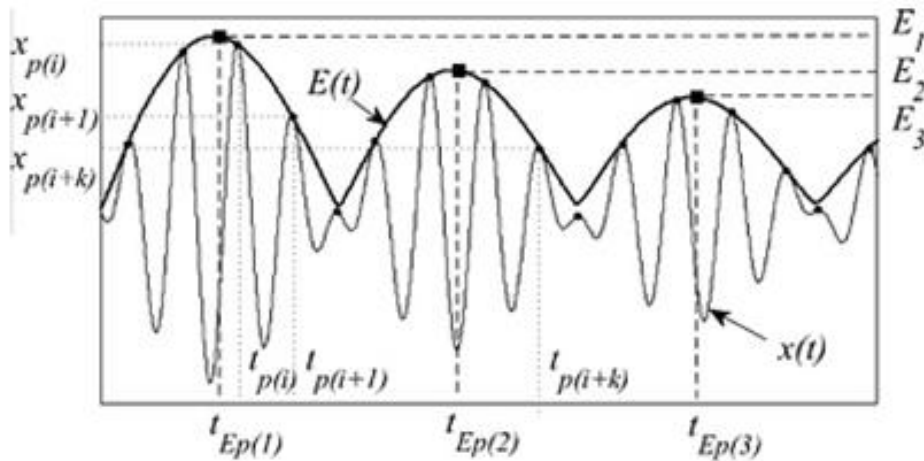


Figure 3. Exponentially decaying vibrations having superimposed amplitude beat

Therefore, we set two goals for our investigation:

Evaluate errors of the LD measurement neglecting presence of amplitude beat and compare them against the resolution of the LD specification in civil engineering practice.

Suggest a LD estimation method that would reduce LD errors to acceptable range.

### 3. Methodology

To estimate LD measurement errors we use computer simulation including the following steps:

Choose a value of  $LD\delta^*$ . It is related to the damping ratio  $\zeta_1^*$  according to the equation is given in [9].

$$\delta^* = \delta_1^* = \frac{2\pi\xi_1^*}{\sqrt{1-\xi_1^{*2}}} \approx 2\pi\xi_1^* \quad (8)$$

Generate a sampled signal  $x(t_i)$  according to Eq.(3)

Obtain an estimated  $LD\delta^*$  using later considered methods.

Estimate LD error.

$$\Delta\delta = \delta^* - \delta_m \quad (9)$$

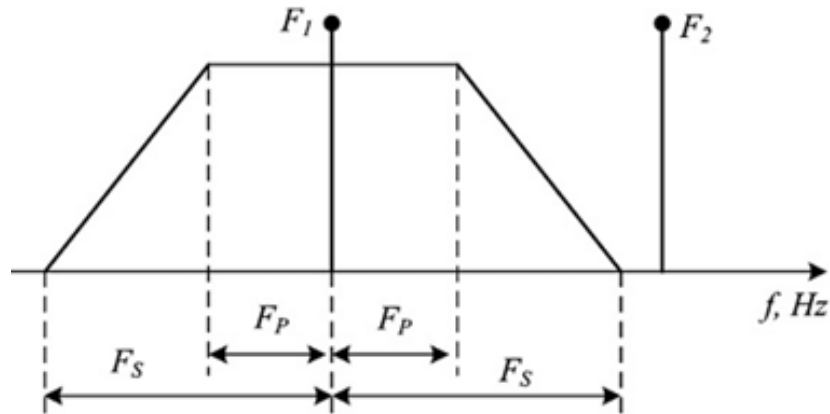
Bellow we specify the investigated LD measurement methods and present LD estimation errors together with the comments on them.

### 4. Results

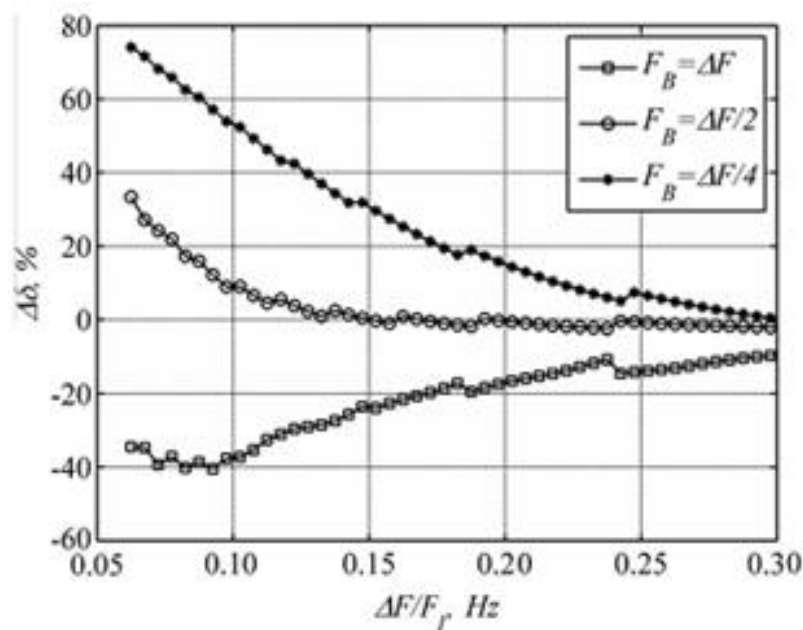
#### 4.1. Standard method based on band-pass filtering

Following the standard LD estimation procedure we apply digital band-pass filter to damp spectral component  $F_2$ . Then Eq.(6) or Eq.(7) is used to calculate  $LD\delta^*$  from filtered time domain waveform. Types of filters investigated include Butterworth, Elliptic and Chebyshev. Since smaller LD estimation errors were observed using Butterworth filter we further present only results calculated with this type of filter. The nature of dependencies given bellow was similar compared to the cases of Elliptic and Chebyshev filters.

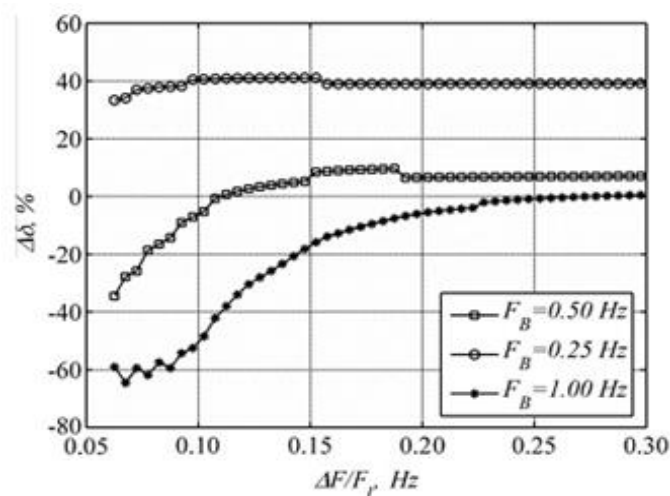
To explore the impact of various combinations of filter parameters upon  $\Delta\delta$  we construct filter's pass-band width  $F_p$  and stop-band width  $F_s$  (see Fig. 4) according to expressions



**Figure 4. Band-pass filter transfer function**



**Figure 5. LD error vs. relative frequency difference (parameter FB is changing in accordance to  $\Delta F, RS=10\text{dB}, \delta=0.10$ )**



**Figure 6. LD error vs. relative frequency difference (parameter FB is constant,  $RS=10\text{dB}, \delta \neq 0.10$ )**

$$F_p = F_B / 2 \quad (10)$$

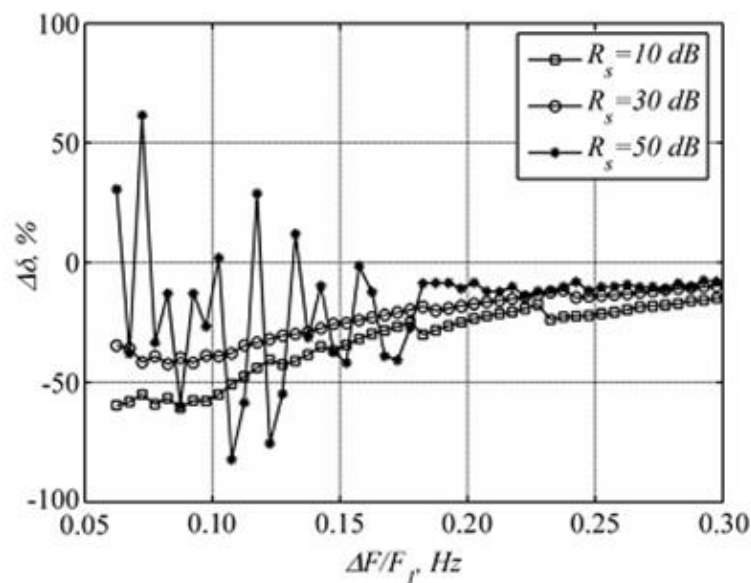
$$F_s = F_p + F_B \quad (11)$$

Where  $F_B$  is the frequency band which was either set in some relationship to the harmonics frequency distance  $\Delta F$  or was set to a fixed value as specified later. Dependencies of LD estimation errors upon the relative harmonic frequency distance  $\Delta F/F_1$  are plotted in Fig. 5 through Fig. 8.

Used filter attenuation  $R_s$  at the stop-band frequency and used  $LD\delta^*$  values are given in the captions of corresponding figures.

Central frequency of the band-pass filter was always set equal to  $F_1$ , i.e. equal to the frequency of the spectral component of interest. Its frequency can be evaluated using Fourier analysis. All plots were obtained at  $F_1 = 8.0$  Hz. Though not proved mathematically we observed the similar error trends in the case  $F_1 \neq 8.0$  Hz.

From the results in the Fig. 5 we can conclude that there exists some optimal frequency band  $F_B$  ensuring the smallest LD errors.



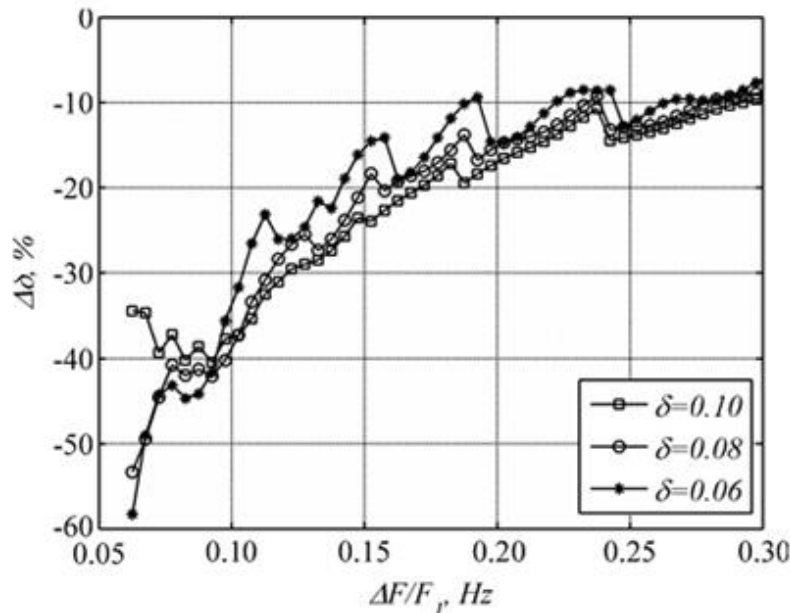
**Figure 7. LD error vs. relative frequency difference (parameter  $F_B$  is changing in accordance to  $\Delta F$ ,  $\delta^* = 0.10$ )**

Selecting either too big (case  $F_B = \Delta F$ ) or too narrow (case  $F_B = \Delta F/4$ ) pass-band and stop-band frequencies result in significant error increase especially in the range  $\Delta F/F_1 < 0.20$ . Any formal procedure to determine optimal  $F_p$  and  $F_s$  is not considered in publications dealing with LD estimation. Obviously, selecting  $F_p$  and  $F_s$  is of less concern when  $\Delta F/F_1 > 0.20$ , i.e. when  $F_1$  and  $F_2$  are not close to each other. The explanation for why  $\Delta\delta$  tends to increase by narrowing  $F_s$  may be related to the influence of longer impulse response of the filter that affects the shape of filtered waveform in time domain.

To provide more evidence about the challenges during filter parameter selection we present Figs. 6 and 7. In the Fig. 6 dependence  $\Delta\delta$  vs.



$\Delta F/F_1$  is given by setting fixed FB irrespective to  $\Delta F$ . There can be stated the problem of the resembling character to the discussed above – by reducing FB we reduce  $\Delta\delta$ , but reducing it too much imposes additional significant errors. Results in the Fig. 6 indicate that the same difficulties can be faced in selecting filter's stop-band attenuation.



**Figure 8. LD error vs. relative frequency difference for different LD values (RS=10dB).**

In the Fig. 8 LD estimation errors are plotted for different LD values.

The length of time domain waveform for every LD value was chosen to be as long as to contain two peaks of amplitude beat. An influence of real LD value  $\delta$  upon estimation errors can be stated from the Fig. 8.

Due to the very close vicinity of frequencies  $F_1$  and  $F_2$  the selection of band-pass filter parameters becomes a challenge. Because of the complexity of relationship between  $\Delta\delta$  the collection of filter parameters (FS,RS,FP) and parameters of vibration record itself ( $\delta$ ,  $F_1$ ,  $F_2$ ) makes it difficult to give a guidelines for filter parameter selection.

Failing to choose optimal filter parameters will lead to the significant LD estimation errors. Indeed, the resolution of LD or corresponding damping ratio values given in scientific papers and research reports is two or three significant digits which makes little sense if estimation errors exceed several tens of percent [1–5]. The key issue here is that after the field measurements and having only vibration records which are used to estimate LD one will not be able to determine the errors  $\Delta\delta$  as it was possible in case of simulation.

Therefore we assume that this size of errors should not be treated as insignificant and there exists a need for better method to estimate LD in described circumstances.

#### 4.2. Modified method

The signal amplitude beat causes significant LD estimation errors as it was shown in the previous section. The idea of proposed method is to take into consideration the beat presence during the selection of peaks to be used for LD estimation by either Eq.(6) or Eq.(7).

According to Eq.(5), at the time moments  $tEp(n)=(2/\omega A)(\pi/2), n=1,2...3$  the envelope value does not depend upon the slowly varying beat which is characterized by the harmonic oscillations with frequency

FA(see Fig.3).

$$E_n = 2X_1 e^{-\xi_1 \omega_c t_{Ep(n)}} \quad (12)$$

Therefore, we propose to calculate LD using expression

$$\delta m = \frac{1}{k} \ln \frac{E_{(n+1)}}{E_{(n)}} \quad (13)$$

Where  $K=FC/FA$  is the number of high frequency oscillation periods between envelope peak moments  $t_{Ep(n)}$  and  $t_{Ep(n+1)}$ .

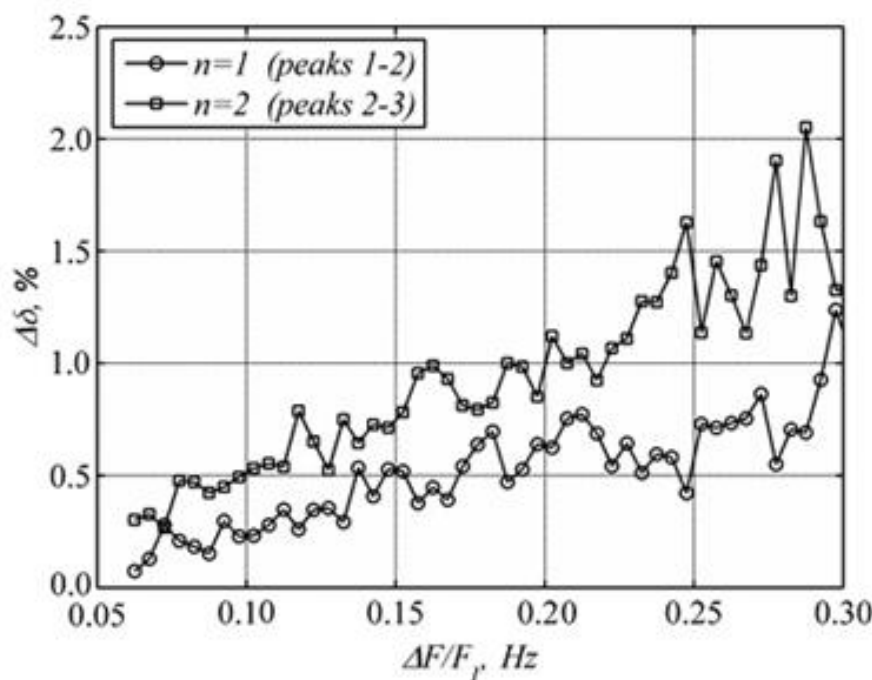
The coefficient  $K$  can be a non-integer.

For the LD estimation using least squares fit method (Eq.(7)) we propose to use only reference points  $(t_{Ep(n)}, E_n)$ .

The obtained results are shown in Figs. 9 and 10.

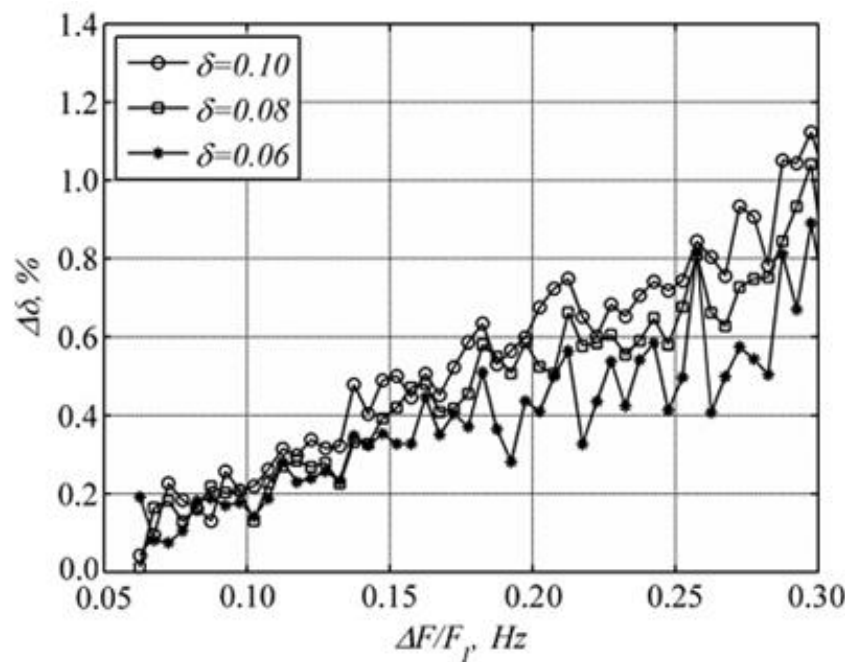
It can be clearly seen that LD estimation errors are by the order smaller compared to the filtering method when frequencies  $F_1$  and  $F_2$  differ less than by (10–20) %. When the difference of frequencies goes beyond that range errors tend to increase and classical filtering method can be applied instead of the proposed method. The Table 1 resumes dependencies presented in Figs.5–8. The error range indicates the interval of LD estimation errors which is valid in case of processing parameters (FB, FS and RS) are within analyzed boundaries (shown in legends and captions of Figs. 5–9).

The difference of the proposed method compared to the filtering method is that it does not require any narrow band-pass filter for attenuation of the close frequency spectral components.



**Figure 9. LD error vs. relative frequency difference of the proposed method.**

LD is estimated using Eq.(13).



**Figure 10. LD error vs. relative frequency difference of the proposed method.**

LD is estimated using least squares fit of three reference points.

Thus, it is free of any filter parameters selection difficulties. We have shown that LD estimation errors are very sensitive to the parameters of band-pass filter when a vibration record contains very close spectral components. Instead of this our method needs vibration record envelope detection and selection of beat peaks on the envelope curve. Vibration record spectrum is used to estimate frequencies of the close spectral components and the beat frequency. The beat period is used to define the location of the second and third beat peaks. Finally, both classical filtering and our proposed method utilize Eqs.(1), (2) to calculate LD estimate.

Another well known LD estimation method is so called random decrement technique[6]. It is hardly applicable in case of bridge excitation by a passing train, because a set of vibration records need for synchronous averaging cannot be produced.

#### 4.3. Testing with bridge vibration record

Both filtering and proposed methods were applied to estimate LD from the vibration record shown in Fig. 2. By alternating filter parameters and peak points used to calculate LD according to Eq.(6)( $k=1$ ) it were obtained LD estimates scattered in the range  $\delta=0.07$ – $\delta=0.11$ .

Then it is difficult to select which one of these estimates should be taken as final. After applying the proposed method based on least squares fit method to approximate Eq.(7) using five reference points (envelope ( $tEp(n), En$ ),  $n=1,5$ ) we received the estimate  $\delta=0.10$ . As can be concluded from the initial testing with real vibration record, the results produced by the modified method do not contradict with the results of filtering method.

**Table 1. Comparison of the method specific errors**

Method	Error range of LD estimation		
	$\Delta F/F_1 \leq 0.10$	$0.10 < \Delta F/F_1 \leq 0.20$	$0.20 < \Delta F/F_1 \leq 0.30$
Filtering (Figs.5–8)	60%/+70%	-50% / +50%	-20% / +40%
Proposed (Figs.9 and10)	0.0%/0.5%	0.0% / 1.0%	0.0% / 2.0%

## 5. Discussions

Despite the promising results in terms of reduced LD estimation errors the effectiveness of the proposed method was not yet validated in the case of phase shift between F1 and F2 components, in the case of amplitude difference  $X1 \neq X2$ , as well as in the case of additive noise presence in the model described by the Eq. (2).

Considering amplitude difference it is obvious that the larger the difference between X1 and X2 ,the advantage offered by the proposed method will be less noticeable. When one of the components is much larger than the other the situation will resemble to classical LD estimation task and filtering method should work fine. However, quantitative issues will be targeted in our following research.

## 6. Conclusions

We have shown that in the presence of very close spectral components in the vibration records used to estimate logarithmic decrement (LD) of bridges the selection of band-pass filter for damping one of the components is difficult. Failing to choose the optimal filter parameters may cause significant LD errors.

The proposed modified LD estimation method does not require band-pass filtering but takes into account presence of amplitude beat in vibration records. Simulation has shown that this method allows reducing LD estimation errors in an order, this way pushing them to the level comparable to the LD resolution usually stated in publications and reports in the field of bridges vibration.

Usefulness of the proposed method in the presence of additive noise and combination of parameters (amplitude ratio, phase) of the adjacent spectral components is still to be investigated.

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